

Exercise 25

Find the limit or show that it does not exist.

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x + 3x^2}}{4x - 1}$$

Solution

Evaluate the limit using the limit laws after factoring out the highest power of x in the denominator.

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sqrt{x + 3x^2}}{4x - 1} &= \lim_{x \rightarrow \infty} \frac{\sqrt{x + 3x^2}}{x \left(4 - \frac{1}{x}\right)} \\ &= \lim_{x \rightarrow \infty} \frac{\frac{1}{x} \sqrt{x + 3x^2}}{4 - \frac{1}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{1}{x^2}(x + 3x^2)}}{4 - \frac{1}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{1}{x} + 3}}{4 - \frac{1}{x}} \\ &= \frac{\lim_{x \rightarrow \infty} \sqrt{\frac{1}{x} + 3}}{\lim_{x \rightarrow \infty} \left(4 - \frac{1}{x}\right)} \\ &= \frac{\sqrt{\lim_{x \rightarrow \infty} \left(\frac{1}{x} + 3\right)}}{\lim_{x \rightarrow \infty} \left(4 - \frac{1}{x}\right)} \\ &= \frac{\sqrt{\lim_{x \rightarrow \infty} \frac{1}{x} + \lim_{x \rightarrow \infty} 3}}{\lim_{x \rightarrow \infty} 4 - \lim_{x \rightarrow \infty} \frac{1}{x}} \\ &= \frac{\sqrt{0 + 3}}{4 - 0} \\ &= \frac{\sqrt{3}}{4} \end{aligned}$$